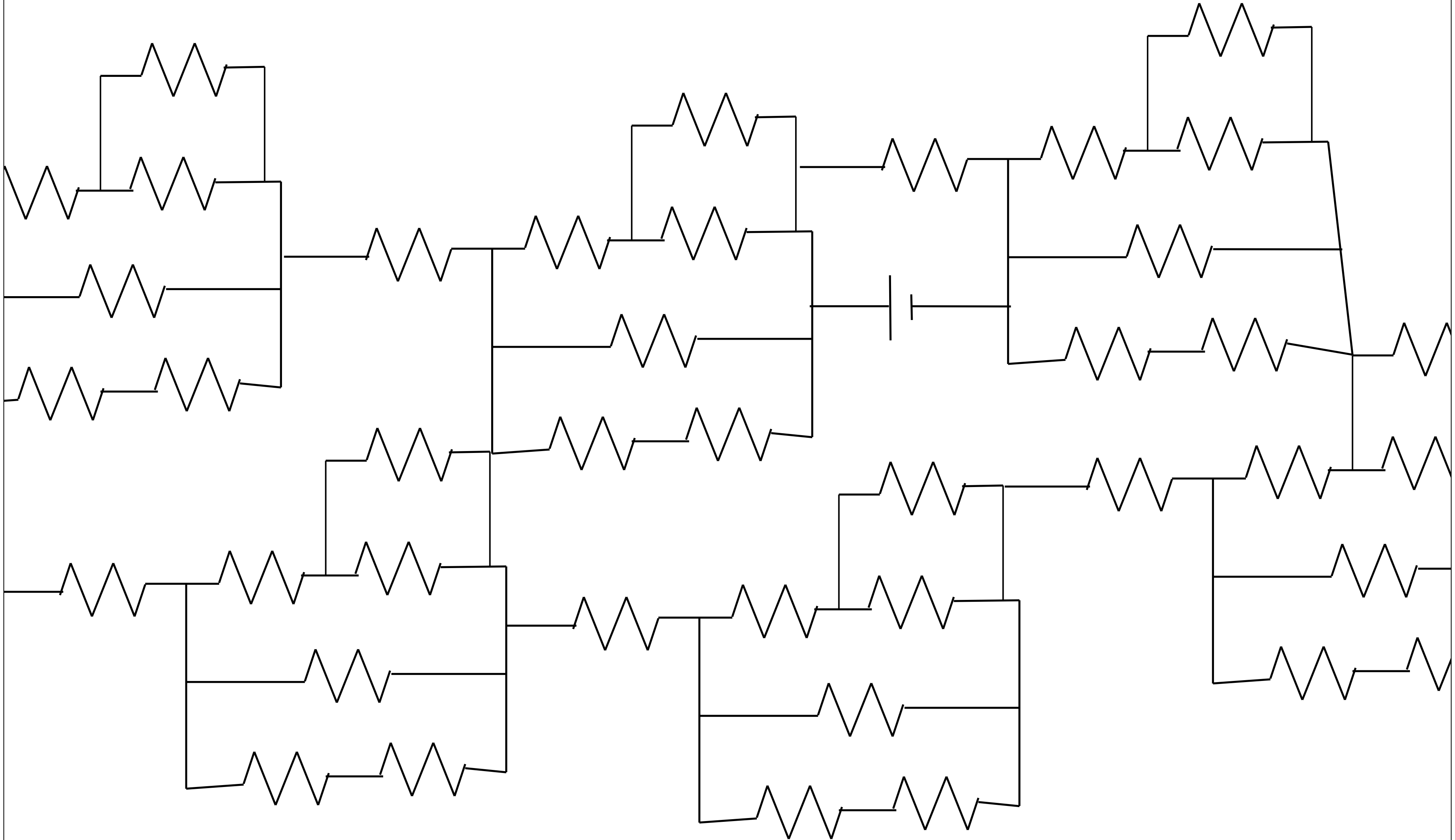
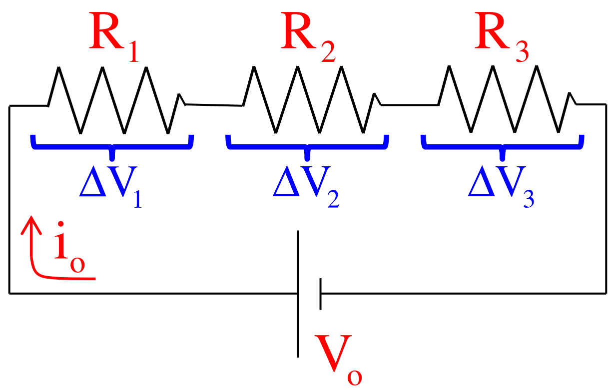
CHAPTER 28A
Elementary DC Electrical Circuits

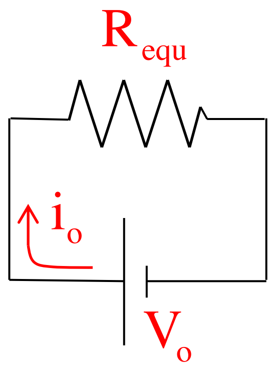


Series Resistor Combinations

Example 2: Derive an expression for the *equivalent resistance* (the single resistor that can take the place of the resistor combination) for the series combination shown to the right. Assume an ideal power supply with no internal resistance.



The idea behind R_{equ} is to find the single resistor that can take the place of all the resistors in the system. In other words, the single resistor that, when put across V_o will draw i_o.



Using the idea that the sum of the voltage drops across all the resistors will equal the voltage drop across the power supply, and including Ohm's Law in the mix, we can write:

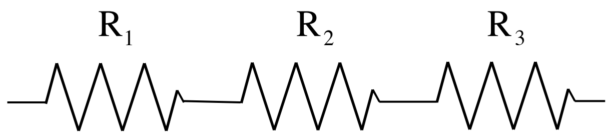
$$V_o = \Delta V_i + \Delta V_2 + \Delta V_3$$

$$\cancel{i_o} R_{eq} = \cancel{i_o} R_1 + \cancel{i_o} R_2 + \cancel{i_o} R_3$$

$$\Rightarrow \quad R_{eq} = R_1 + R_2 + R_3$$

Characteristics of a Series Combinations

--Each element in a series combination is attached to its neighbor in *one place only*.



--Current is common to each element in a series combination.

--There are no nodes (junctions—places where current can slit up) internal to series combinations.

--The equivalent resistance for a series combination is: $R_{eq} = R_1 + R_2 + R_3 + \ldots$

--This means the equivalent resistance is larger than the largest resistor in the combination;

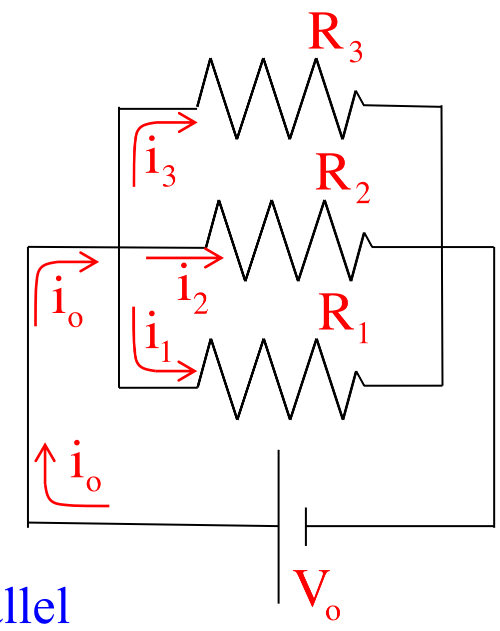
--This means that if you add a resistor to the combination, R_{eq} will *increase* and the current through the combination (for a given voltage) will *decrease*.

Example 3: What's the equivalent resistance of a 5 Ω, 6 Ω and 7 Ω resistor in series?

$$R_{eq} = (5\Omega) + (6\Omega) + (7\Omega)$$
$$= 18\ \Omega$$

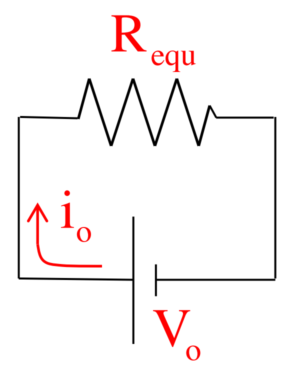
Parallel Resistor Combinations

Example 3: Derive an expression for the *equivalent resistance* (the single resistor that can take the place of the resistor combination) for the parallel combination shown to the right. Assume an ideal power supply with no internal resistance.



What's common in a parallel combination is the voltage drop across each element.

Also, in this case, the sum of the currents through the parallel combination must equal the current drawn from the power supply. Using that and Ohm's Law, we can write:

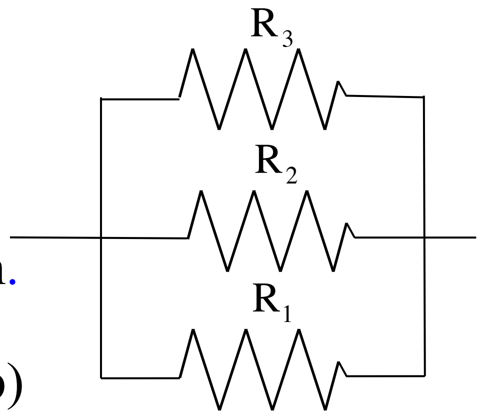


$$i_o = i_i + i_2 + i_3$$

$$\frac{\cancel{V}_o}{R_{eq}} = \frac{\cancel{V}_o}{R_1} + \frac{\cancel{V}_o}{R_2} + \frac{\cancel{V}_o}{R_3}$$

$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Characteristics of a Parallel Combinations



--*Each element* in a series combination is attached to its neighbor in *two place*.

--*Voltage* is common to each element in a parallel combination.

--*There are* nodes (junctions—places where current can slit up) internal to parallel combinations.

--*The equivalent resistance* for a parallel combination is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

--*This means* the equivalent resistance is SMALLER than the smallest resistor in the combination;

--*And,* if you add a resistor to the combination, R_{eq} will *decrease* and the current through the combination (for a given voltage) will *increase*.

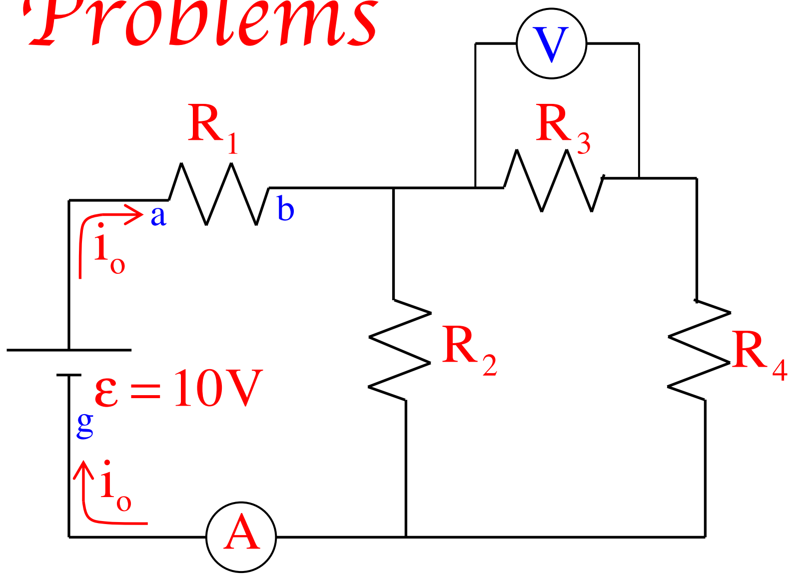
Example 3: What's the equivalent resistance of three one-ohm resistors in parallel?

$$1/R_{eq} = 1/(1\Omega) + 1/(1\Omega) + 1/(1\Omega)$$

$$\Rightarrow 1/R_{eq} = 3 \Rightarrow R_{eq} = .333\ \Omega$$

Seat of the Pants Problems

Example 4: An *ideal power supply* has EMF $\varepsilon = 10V$ powering it. Any blue letters that show up designate points on the circuit. Assume the *low voltage terminal* of the p.s. is at zero volts. Assume the resistor values are the same as the resistor subscripts.



a.) What is the first thing you would do if asked to work with this circuit?

Redraw the circuit without the meters. They aren't doing anything in the circuit except identifying a branch or resistor for which you want a current.

b.) What is the *absolute electrical potential* at *Point a*?

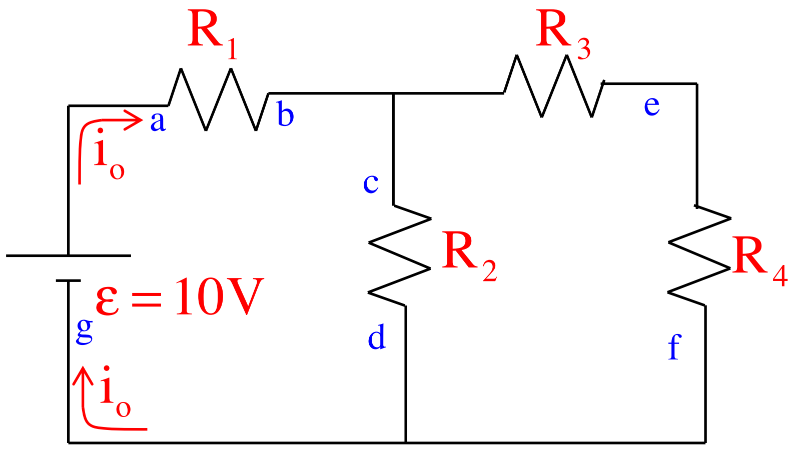
There is essentially no resistance between *Point a* and the high voltage terminal of the p.s., so their voltages are the same point being $V_a = 10v$.

c.) What will the ammeter read?

Some amount of current is being drawn from the power supply. Being in the same branch as *Points a*, *b* and *g*, it will be the same for all three points. It will also be the current through the ammeter. So how do we get that?

c.) ammeter?

Here is the circuit with the meters removed. The trick here is to find the equivalent resistance for the circuit, then use Ohm's Law.

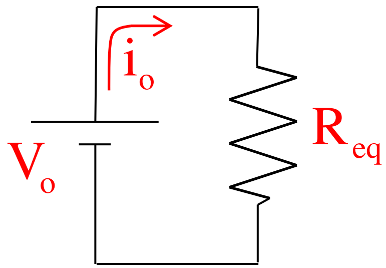


This circuit is R_1 in series with R_2 in parallel with R_3 and R_4 in series. That is:

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3 + R_4} \right)^{-1}$$

$$= (1\ \Omega) + \left(\frac{1}{(2\ \Omega)} + \frac{1}{(3\ \Omega) + (4\ \Omega)} \right)^{-1}$$

$$= 2.56\ \Omega$$

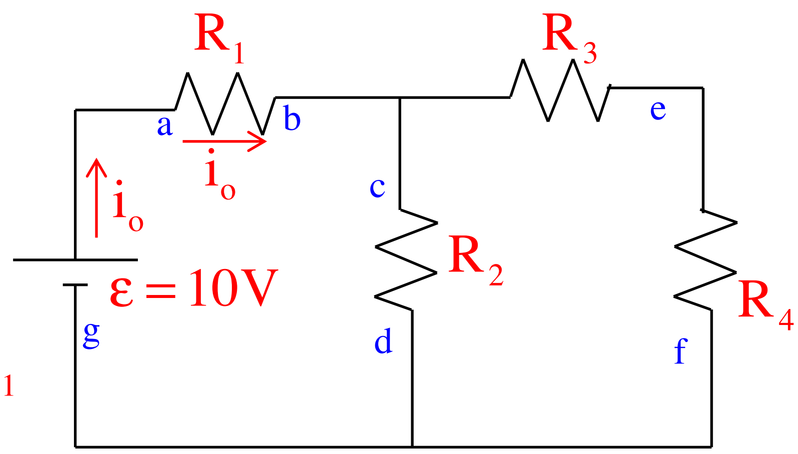


Using the R_{eq} circuit: $V_o = i_o R_{eq} \Rightarrow i_o = \frac{V_o}{R_{eq}}$

$$= \frac{(10\ V)}{(2.56\ \Omega)} = 3.9\ A$$

The ammeter will read 3.9 amps.

d.) How much power does R_2 dissipate?



We know the absolute electrical potential (the voltage) at *Point a* is 10 volts.

We know current goes from *high voltage* to *low voltage*, so the *voltage change* across R_1 must be a voltage DROP equal to:

$$\Delta V_1 = i_o R_1$$
$$= (3.9\ \text{A})(1\ \Omega) = 3.9\ \text{V}$$

Logic dictates that the absolute electrical potential at *Point b is*:

$$V_b = V_a - \Delta V_1$$
$$= (10\ \text{V}) - (3.9\ \text{V})$$
$$\Rightarrow \quad V_b = 6.1\ \text{V} \quad (= V_c)$$

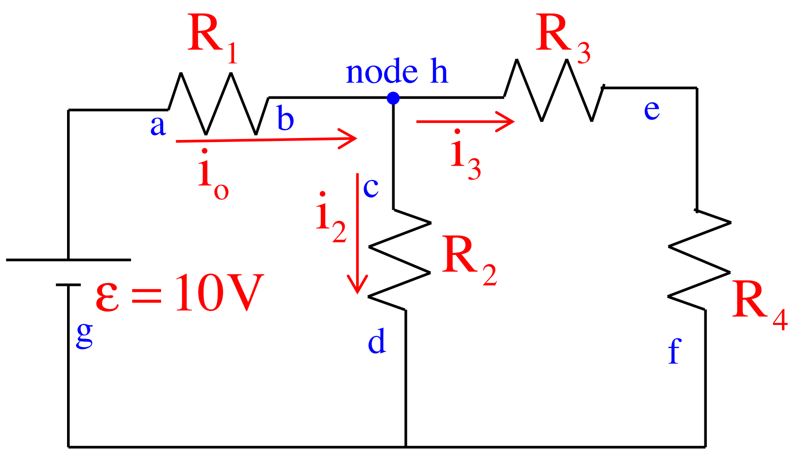
Because the absolute electrical potential at *Point d* is *zero*, the voltage across R_2 equals $V_c = 6.1\ \text{V}$ and:

$$V_2 = i_2 R_2$$
$$\Rightarrow \quad (6.1\ \text{V}) = i_2 (2\ \Omega)$$
$$\Rightarrow \quad i_2 = 3.05\ \text{A}$$

The power dissipated by R_2 is, then:

$$\Rightarrow \quad P_2 = (i_2)^2 R_2$$
$$= (3.05\ \text{A})^2 (2\ \Omega)$$
$$= 18.6\ \text{W}$$

e.) What does the voltmeter read?



You should begin to see a pattern here. Every question, whether it be asking for an *ammeter reading* or *voltmeter reading* or *power calculation* or *current through an element* or *voltage cross an element*, they all require you to determine the CURRENT through the branch in which the element exists. That, in general, is what you will always be doing—trying to derive expressions for current values.

In this case, you could determine the current through the far-right branch (so you could use Ohm's Law on R_3 to get what the voltmeter would read) by using the same approach we used to get the current in the central branch in Part c (you'd just be using *Points e* and *f* instead of *Points c* and *d* in the process).

Or . . .

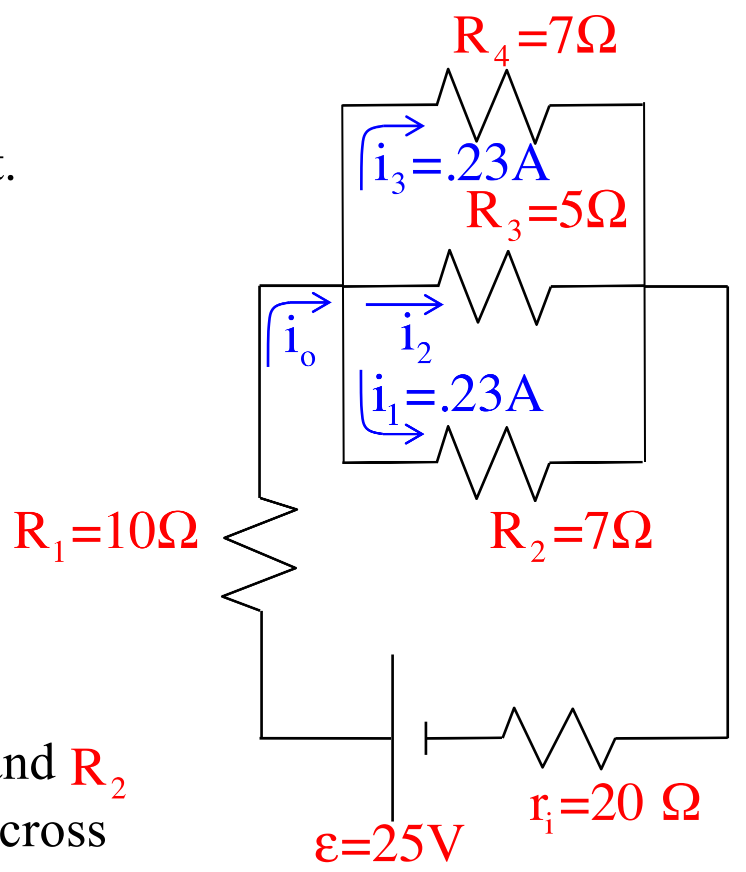
Look at node h

$$i_o = i_2 + i_3$$
$$\Rightarrow i_3 = i_o - i_2$$
$$= 3.9\text{A} - 3.05\text{A}$$
$$= .85\text{A}$$

So

$$V_{meter} = i_3 R_3$$
$$= (.85\text{A})(3\ \Omega)$$
$$= 2.55\ \text{V}$$

Example 5: A power supply with 20 Ω of internal resistance is used to power a circuit. If the current through R_4 is .23 amps, what is the current through R_1?



Start with what is obvious.

The current through R_1 in the bottom branch will equal all the currents in the parallel combination put together, or

$$i_1 = i_2 + i_3 + i_4$$

We know the current through R_4 (given) and R_2 (same size resistance with same voltage across it), so all we need is the current through R_3.

The voltage across each of the parallel resistors is the same, and equal to:

$$V_4 = i_4 R_4 = (.23\text{A})(7\ \Omega) = 1.61\text{ V}$$

The the current through R_3 is:

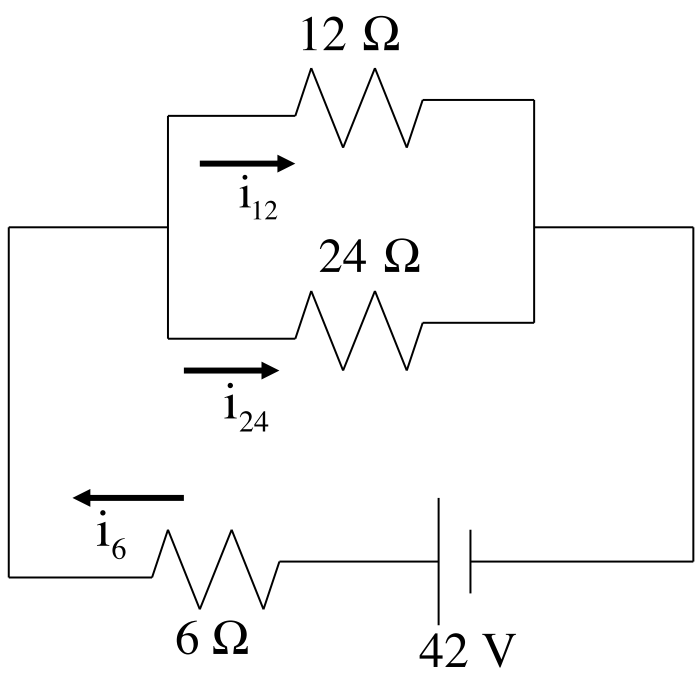
$$V_3 = i_3 R_3$$

$$(1.61\text{ V}) = i_3 (5\ \Omega)$$

$$\Rightarrow i_3 = .32\text{ A}$$

So: $i_1 = .23\text{A} + .32\text{A} + .23\text{A} = .78\text{A}$

Example 6: The current from the battery is 3 amps. How much current goes through the upper branch of the parallel combination?



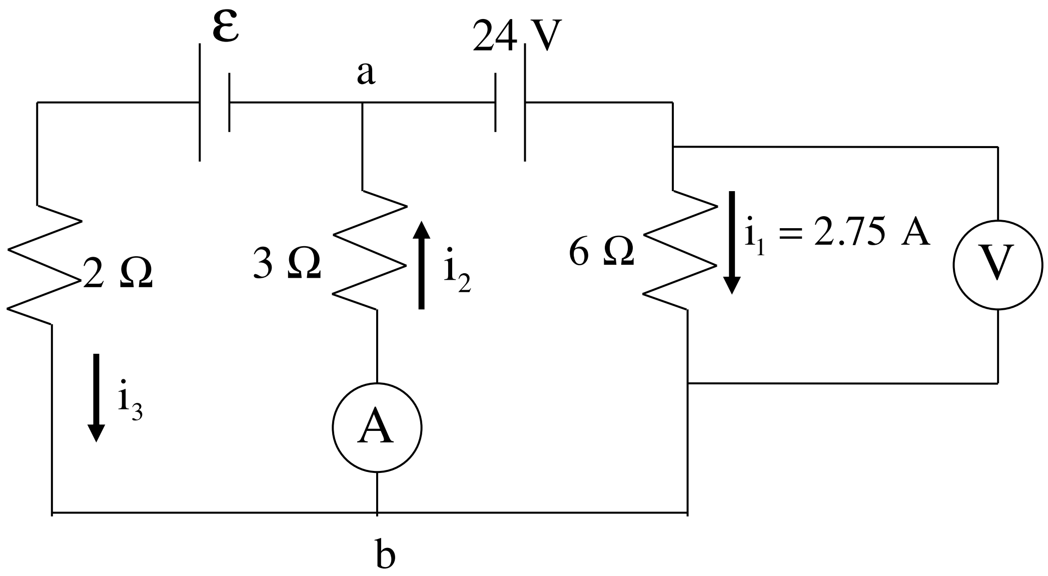
This is another use-your-head question.

If the upper branch has half the resistance of the lower branch, it should draw twice the current.

With 3 amps coming in, that means 2 amps should pass through the upper branch.

Note: AP questions often have easy, non-mathematical, use-your-head solutions like this. That is why I'm showing you screwball problems like this. We will get into a more formal approach for analyzing circuit problems shortly.

Example 7: Consider the circuit shown.



a.) What does the voltmeter read?

$$V = i_1 R$$
$$= (2.75\text{A})(6\ \Omega)$$
$$= 16.5\ \text{V}$$

b.) What is the voltage difference between *Points a* and *b*?

Assuming the voltage at *Points a* is zero (not a bad bet as it's sandwiched between two battery ground terminals), the voltage changes will be due to the increase due to the battery in the right branch and the drop due to the 6 ohm resistor. That is:

$$V_{ab} = 24 - (2.75\text{A})(6\ \Omega)$$
$$= 7.5\ \text{V}$$

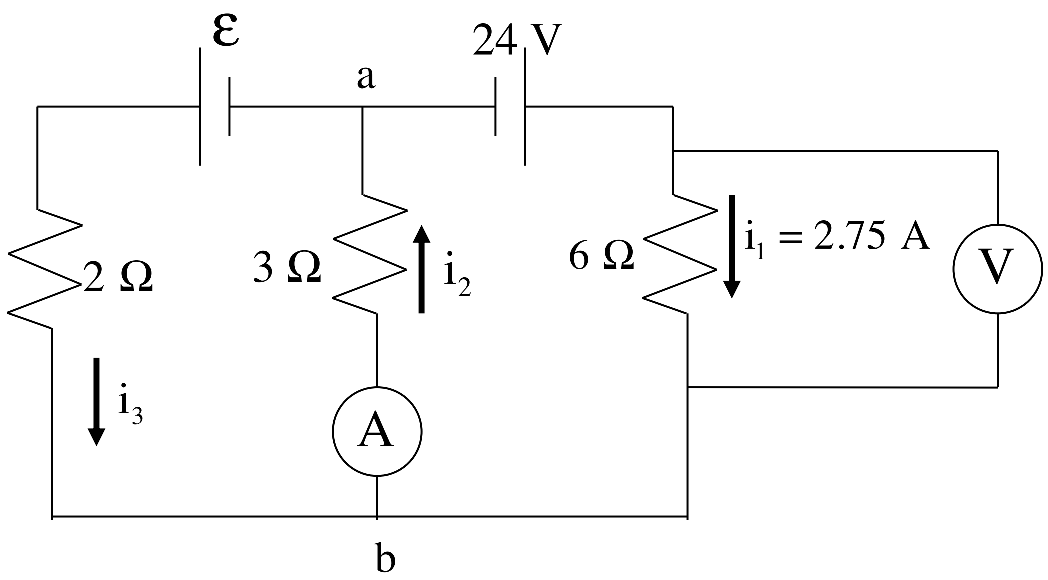
c.) What does the ammeter read?

This is just the *current* through the 3 ohm resistor, or:

$$V_{ab} = i_2 R_3$$
$$7.5\text{V} = i_2 (3\ \Omega)$$
$$\Rightarrow \quad i_2 = 2.5\ \text{A}$$

c.) What is the current i_3?

The currents into node a have to equal the currents out of node a, so we can write:



$$i_2 = i_1 + i_3$$

$$\Rightarrow \quad i_3 = i_2 - i_1$$

$$= (2.5) - (2.75)$$

$$= -0.25 \text{ amps}$$

b.) What is the battery EMF in the left branch?

Using the same technique of tracking voltage changes starting at Point a, remembering that the voltage DROPS if you are traversing in the direction of the current flow and using the negative value for i_3, we can write:

$$\varepsilon - 2i_3 = V_{ab}$$

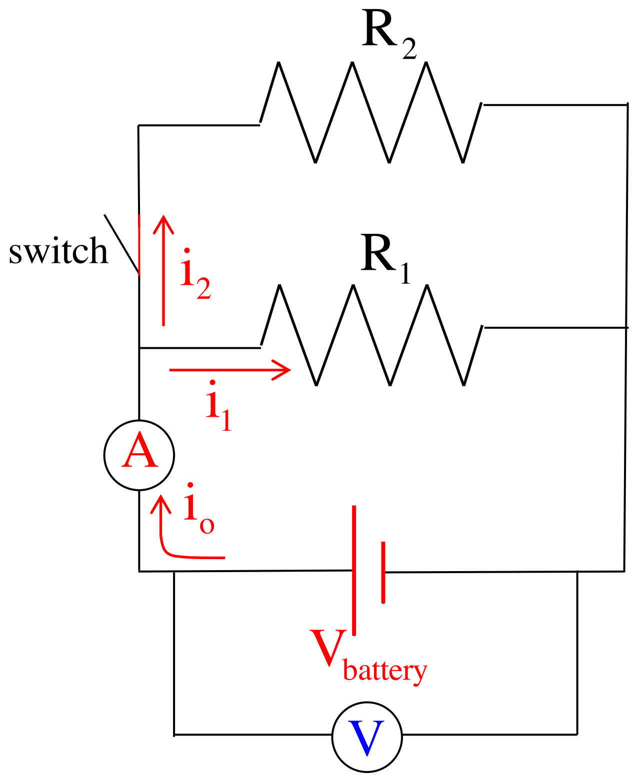
$$\Rightarrow \quad \varepsilon = 2i_3 + V_{ab}$$

$$= 2(-0.25 \text{ A}) + (7.5 \text{ v})$$

$$= 7.0 \text{ v}$$

EMF and Terminal Voltage

Example 8: Consider the circuit to the right. If the resistors represent light bulbs:



a.) What does the *ammeter* read when the switch is open? (step 1—redraw without the meters)

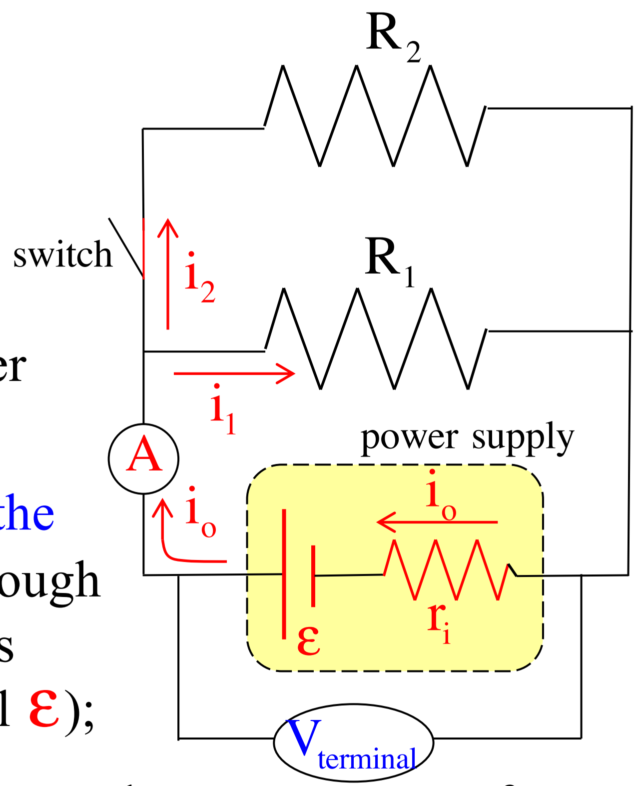
All the battery's voltage drop happens across the resistor R_1, and the current through the ammeter is just i_1, so:

$$V_{bat} = i_1R_1 = i_oR_1 \quad \Rightarrow \quad i_o = \frac{V_{bat}}{R_1}$$

b.) In an ideal world, what should happen to the current through R_1 when the switch is thrown?

Nothing, and this is tricky. If you think of the current from the battery as fixed, you'll conclude R_1's current will drop as some current will now be needed for R_2. But au, contraire. Current through R_1 is governed by the voltage across it and its resistance, neither of which changes when you throw the switch. So what happens? The battery outputs MORE CURRENT so $i_o = i_i + i_2$ and R_1, in theory, should remain unaffected.

c.) Here's the rub: if you carry this experiment out in the real world, the light bulb associated with R_1 will *actually dim* suggesting that the current through R_1 has diminished. So what's going on?



The problem lies in the internal workings of power supplies. There are actually two parts to a p.s.:

1.) There is the part of the battery that creates the electric field that motivates charge to move through the wires. This part has the units of *volts* and is called *the electromotive force,* or EMF (symbol ε);

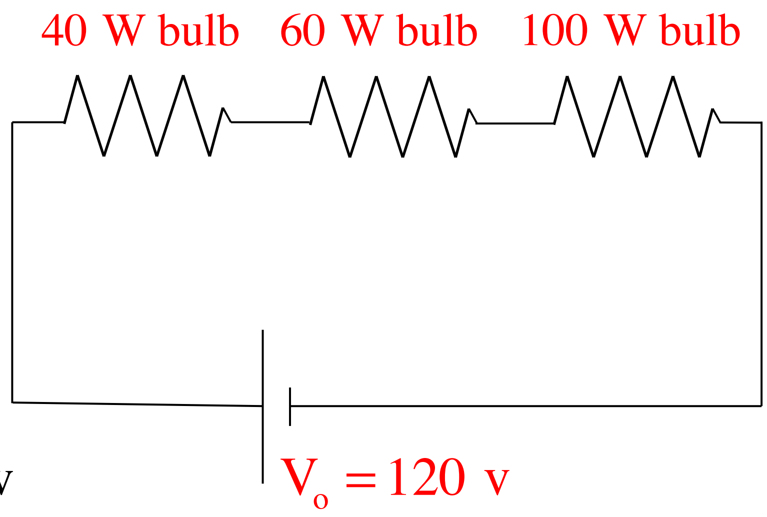
2.) Although it is possible to "rectify" a power supply to compensate for this, a power supply in its natural state also has internal resistance r_i.

Including the voltage drop due to r_i, this means a VOLTMETER will read what is called *the terminal voltage* (i.e., the voltage as measured at the terminals of the p.s.) equal to: $V_{terminal} = \varepsilon - i_o r_i$

Soooo, if you increase i_o by throwing the switch, $V_{terminal}$ does DOWN across the resistors and the bulbs will dim!

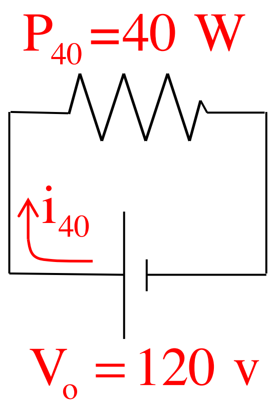
Power and Resistors in Series

Example 9: Consider a 40 watt, a 60 watt and a 100 watt lightbulb hooked up in series across a 120 volt (RMS) power supply (a wall socket). Which of the bulbs will shine the brightest? Justify your prediction.



The brightest will be the 40 watt bulb. How so?

The first thing we need to know is how much current is required to make each bulb glow to its maximum capacity. To do that, consider each bulb by itself across a standard 120 volt (RMS) outlet:

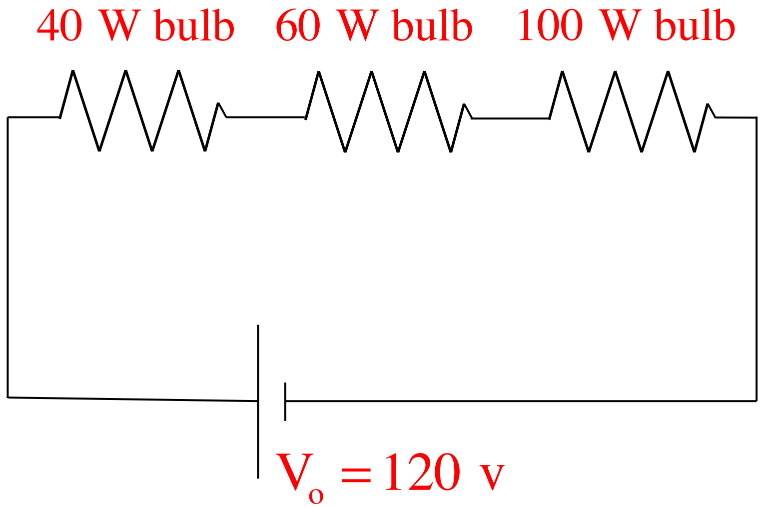


$$P_{40} = i_{40} V_o$$

$$\Rightarrow \quad (40 \text{ W}) = i_{40} (120 \text{ v})$$

$$\Rightarrow \quad i_{40} = .33 \text{ A}$$

Executing a similar process for the other two bulbs yields:



$$P_{60} = i_{60}V_o$$

$$\Rightarrow \quad (60\ \text{W}) = i_{60}(120\ \text{v})$$

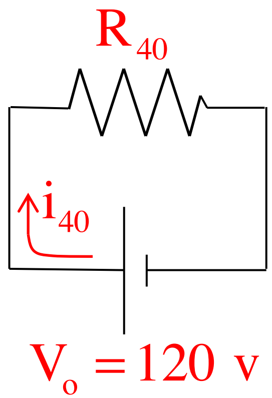
$$\Rightarrow \quad i_{60} = .50\ \text{A}$$

$$P_{100} = i_{40}V_o$$

$$\Rightarrow \quad (100\ \text{W}) = i_{100}(120\ \text{v})$$

$$\Rightarrow \quad i_{100} = .83\ \text{A}$$

The next thing we need to know is the resistance of each bulb. Using Ohm's Law:



$$V_0 = i_{40}R_{40}$$

$$\Rightarrow \quad (120\ \text{v}) = (.33\ \text{A})R_{40}$$

$$\Rightarrow \quad R_{40} = 360\ \Omega$$

Executing a similar process for the other two bulbs yields:

$$V_0 = i_{60}R_{60}$$

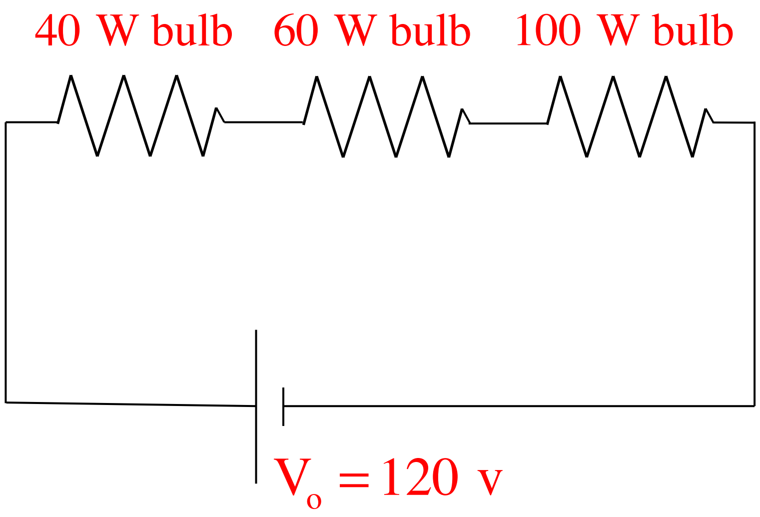
$$\Rightarrow \quad (120\text{ v}) = (.5\text{ A})R_{60}$$

$$\Rightarrow \quad R_{60} = 240\ \Omega$$

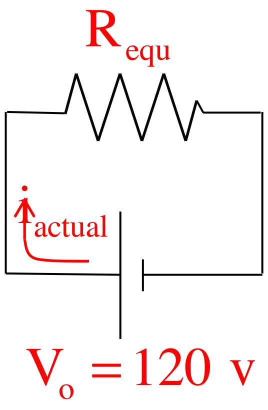
$$V_0 = i_{100}R_{100}$$

$$\Rightarrow \quad (120\text{ v}) = (.83\text{ A})R_{100}$$

$$\Rightarrow \quad R_{100} = 145\ \Omega$$



With that, the equivalent resistance and, hence, actual current in the circuit becomes:



$$R_{eq} = 360\Omega + 240\Omega + 145\ \Omega$$
$$= 745\Omega$$

and

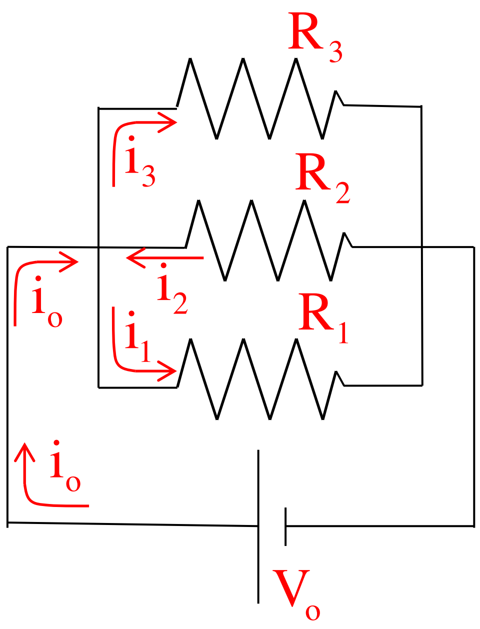
$$V_o = i_{actual}R_{eq}$$

$$\Rightarrow \quad (120\text{ v}) = i_{actual}(745\ \Omega)$$

$$\Rightarrow \quad i_{actual} = .16\text{ A}$$

which is way too little current to power the 100 W bulb and just barely enough to power the 40 W bulb!

Sign of a Current



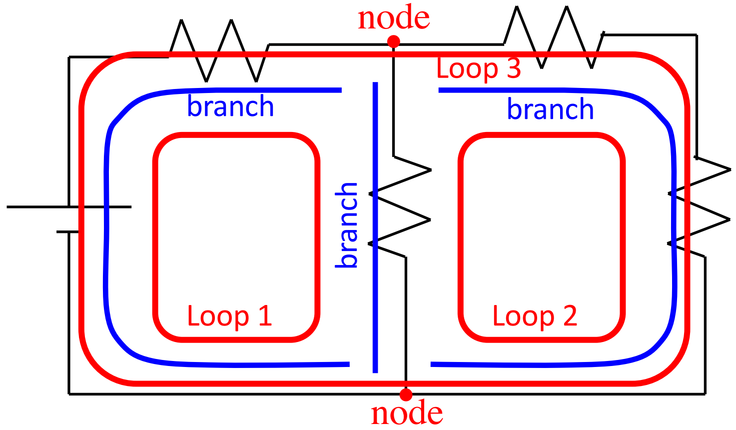
Example 10: Let's say you were to solve for the current through the middle branch of the parallel combination, and the numerical answer turned out to be -8 amps.

What would the negative sign signify?

It would tell you you had assumed the WRONG DIRECTION for charge flow in the branch, which is to say, the arrow denoting the current in the branch would be in the wrong direction.

Consequence: You could state this fact, but you wouldn't have to change anything. The real point of order is that if you were to use that current value in subsequent calculations, you would leave it negative.

Some Definitions



A branch: A section of a circuit in which the current is the same everywhere.

--*elements in series* are a part of a single branch (look at sketch).

--*in the circuit* to the right, there are three branches.

A node: A *junction* where current can split up or be added to.

--*elements* in parallel have nodes internal to the combination.

--*in the circuit* above, there are two nodes.

A loop: Any closed path inside a circuit.

--*in a circuit*, loops can be traverse in a clockwise or counterclockwise direction.

--*in the circuit* above, there are three loops.

For your Amusement

For the circuit to the right:

*a.) How many **branches*** are there?

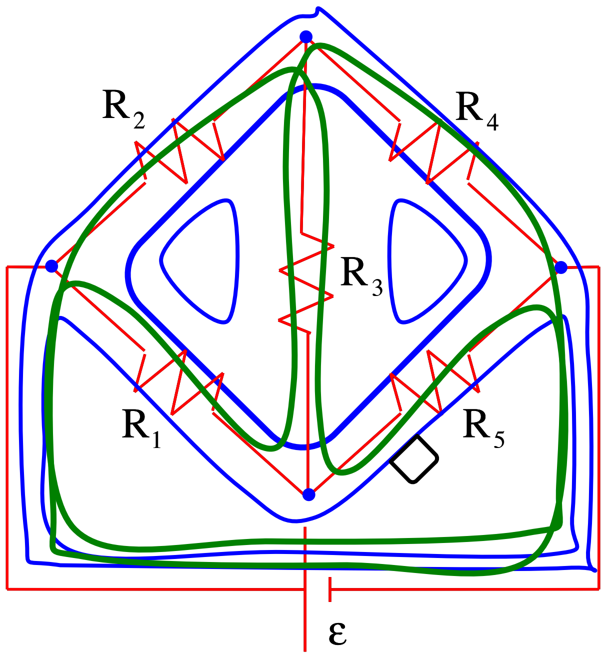
six

*b.) How many **nodes*** are there?

four

*c.) How many **loops*** are there?

seven



And that last little nubbin is supposed to be a tooth, cause this looks like a face to me!